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CORRESPONDENCE FLOWS FOR WINGS IN LINEARIZED POTENTIAL  
FIELDS AT SUBSONIC AND SUPERSONIC SPEEDS

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SUMMARY

The method of relating the solution of a flow field to the solution of a simpler or known boundary-value problem is extended, subject to edge corrections for finite plan forms, to the most general type of prescribed boundary conditions on a wing in the linearized potential field at subsonic and supersonic speeds. It is shown that, subject to edge corrections, the flow field due to an arbitrary prescribed velocity on the wing can be expressed in terms of the flow field due to a uniform prescribed velocity on the wing. The general procedure for determining the edge corrections is indicated. Specific correspondence formulas are derived for rectangular wings at subsonic and supersonic speeds for cases that do not require the solution of an integral equation. Several examples are given for the rectangular wing at supersonic speeds.

INTRODUCTION

One of the well-known methods for solving boundary-value problems is the expression of the solution in terms of the solution for a simpler or known boundary-value problem. The utilization of this method in the classical linearized wing theory has thus far been limited to the case in which two or more prescribed velocity distributions on a wing are related by a constant and the relation is extended to the entire flow field.

In the analysis prepared at the NACA Lewis laboratory and presented herein, the method of relating the solution of a flow field to the solution of a simpler or known boundary-value problem is extended, subject to edge corrections for finite plan forms, to the most general type of prescribed boundary conditions on a wing in the linearized potential field. A generalized rule for correspondence flows is developed that applies to lifting and nonlifting wings at subsonic and supersonic speeds with special consideration required for the edges of finite plan forms. The term "correspondence flows", or "correspondence formulas", is used herein to refer to the relation

between the disturbance parameters of one flow field to those of another flow field. The general procedure for determining the required edge corrections is indicated. Specific correspondence formulas are derived for rectangular wings at subsonic and supersonic speeds for cases that do not require the solution of an integral equation. Several examples of correspondence flows are given for the rectangular wing at supersonic speeds.

### SYMBOLS

The following symbols are used in this report:

|         |                                                                                                                                     |
|---------|-------------------------------------------------------------------------------------------------------------------------------------|
| A       | arbitrary constant                                                                                                                  |
| a       | constant used to describe prescribed velocity $q$ on wing<br>(See equation (3a) and subsequent discussion.)                         |
| B =     | $\sqrt{M^2 - 1}$                                                                                                                    |
| c       | wing chord                                                                                                                          |
| h       | wing semispan                                                                                                                       |
| I       | value of variable of integration for which potential<br>function and its derivatives vanish                                         |
| $I_T$   | refers to I limit of integration for flow initiating<br>from wing trailing edge                                                     |
| K       | constant in equation (2) ( $K = 1/2\pi$ at subsonic speeds;<br>$K = 1/\pi$ at supersonic speeds)                                    |
| M       | free-stream Mach number                                                                                                             |
| n, k    | 0, 1, 2, ... $\infty$ ; constants used to describe prescribed<br>velocity on wing (See equation (3a) and subsequent<br>discussion.) |
| $p, q'$ | angular velocities about x- and y-axes, respectively                                                                                |
| q       | disturbance velocity used to replace $u$ or $w$                                                                                     |
| r =     | $\sqrt{(x-\xi)^2 - B^2 [(y-\eta)^2 + z^2]}$                                                                                         |
| u, v, w | disturbance velocities of fluid in flow field along<br>x-, y-, and z-axes, respectively                                             |

|                                     |                                                                                                                                         |
|-------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| $V$                                 | free-stream velocity                                                                                                                    |
| $x, y, z$                           | rectangular coordinates with origin at leading edge of center section (See fig. 1.)                                                     |
| $y_a$                               | y-coordinate measured from leading edge of tip section                                                                                  |
| $\alpha$                            | angle of attack                                                                                                                         |
| $\Delta q_{n,k}, \Delta \phi_{n,k}$ | increment in $q_{n,k}$ or $\phi_{n,k}$ caused by wing cut-offs along edges that form wing boundary                                      |
| $\xi, \eta$                         | auxiliary variables of integration used to replace $x$ and $y$ , respectively                                                           |
| $\tau$                              | region of integration (See discussion following equations (2) and (4).)                                                                 |
| $\bar{\phi}$                        | disturbance parameter in flow field used to replace $\phi$ , $u$ , $v$ , or $w$                                                         |
| $\phi$                              | disturbance-velocity potential on upper surface of airfoil                                                                              |
| $\Omega$                            | function representing solution of linearized partial differential equation for velocity potential                                       |
| Subscripts:                         |                                                                                                                                         |
| $n, k$                              | $0, 1, 2, \dots, \infty$ ; designate flow field due to given prescribed velocity on wing (See equation (3a) and subsequent discussion.) |
| $S, RS, LS$                         | any side edge, right side edge, and left side edge, respectively                                                                        |
| $T$                                 | wing trailing edge                                                                                                                      |
| $u, l$                              | upper and lower surfaces of airfoil, respectively                                                                                       |
| $x, y, z$                           | indicate partial differentiation with respect to subscript variable                                                                     |
| wake                                | contribution of wake                                                                                                                    |

## GENERAL ANALYSIS

## Basic Theory

The analysis considers the derivation of correspondence formulas for nonlifting and lifting wings of arbitrary plan form at subsonic and supersonic speeds. The correspondence formulas derived in the analysis are based on the usual conditions for thin wings in the linearized potential field. A valid solution is therefore required to meet the following conditions:

1. The streamlines of the flow must be tangent to the airfoil surface ( $z=0$ ).

2. (a) For wings with symmetrical profiles at zero lift,  $w(x,y,0) = 0$ , except on wing.

(b) For lifting wings,  $u(x,y,0) = 0$ , except on wing, and  $v(x,y,0) = 0$ , except on wing and in wake.

3. (a) In subsonic flow,  $\phi$  and all its derivatives vanish at infinity.

(b) In supersonic flow,  $\phi$  vanishes at the foremost Mach wave and all its derivatives vanish ahead of this Mach wave.

4. The potential  $\phi$  must satisfy the partial differential equation

$$(1-M^2)\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \quad (1)$$

A solution for the disturbance-flow parameters in the linearized theory can be developed by integrations of source and doublet singularities in the plane of the wing ( $z=0$ ). The basic equation is (See, for example, references 1 and 2.)

$$\Omega(x,y,z) = -\frac{K}{2} \iint_{\tau} \left[ \frac{1}{r} \left( \frac{\partial \Omega_u}{\partial z} - \frac{\partial \Omega_l}{\partial z} \right) + (\Omega_u - \Omega_l) \frac{\partial}{\partial z} \left( \frac{1}{r} \right) \right] d\xi d\eta \quad (2)$$

where the function  $\Omega$  is a solution of equation (1), the region of integration  $\tau$  includes the entire  $z=0$  plane that can influence the point  $x,y,z$ , and

$$r = \sqrt{(x-\xi)^2 - B^2 [(y-\eta)^2 + z^2]}$$

In subsonic flow, the factor  $K$  is equal to  $1/2\pi$ ; in supersonic flow,  $K$  is equal to  $1/\pi$  and only the finite part of the integral must be used. If the preceding boundary conditions 3 are specified, the function  $\Omega$  can represent either the velocity potential or any of its derivatives or integrals. If the derivative or integral of  $\varphi$  becomes infinite, the substitution of this function for  $\Omega$  in equation (2) depends on the condition that the isolation of the singularity yields a finite integrand in the limit.

#### Outline of Method

As is well-known in the linearized theory, if two prescribed velocity distributions on a wing are related by a constant, the relation can be extended to the entire flow field. Thus, if  $q_1$  and  $q_0$  represent two differently prescribed velocity distributions on a wing such that

$$q_1 = Aq_0 \quad (\text{on wing})$$

where  $A$  is a constant, then

$$q_1(x, y, z) = Aq_0(x, y, z)$$

Similarly, the factor of proportionality  $A$  between  $q_1$  and  $q_0$  also applies to the other disturbance parameters in the entire flow field; that is,

$$\varphi_1(x, y, z) = A\varphi_0(x, y, z)$$

and similarly, for the corresponding derivatives of  $\varphi_1$  and  $\varphi_0$ .

In an analogous manner, it is subsequently demonstrated that, for an infinite wing (the term "infinite wing" is used herein to refer to a wing on which all edges are sufficiently distant so that the edge effects can be ignored), if any of the derivatives of a prescribed velocity distribution (for example,  $(q_n)_x$ ) are related on the wing by a constant to another prescribed velocity distribution  $q_{n-1}$  or to any of its derivatives, then the given relation can be extended to all corresponding disturbance parameters in the entire flow field. Furthermore, it is shown that the relation between the disturbance parameter for two different flows or correspondence formulas for wings of finite plan form can be derived by superposing appropriate edge corrections to the formulas obtained for the infinite plan form.

The correspondence formulas in the subsequent development are obtained by integrations of functions representing the disturbance velocity potential or its derivatives between definite limits. In many cases, the lower limit is the value for which the potential function vanishes. (See conditions 3(a) and 3(b) given in the section Basic Theory.) This type of lower limit is represented throughout the derivation by  $I$ . In subsonic flows, the perturbation velocities vanish at the limit  $I$ . In certain types of supersonic flow, however, a finite jump occurs in the perturbation velocities at the  $I$  limit (that is, across the foremost Mach wave), which can be evaluated by simple shock-wave theory. (See, for example, reference 3.)

#### Generalized Form of Arbitrary Prescribed Velocity on Wing

The most important problems in wing theory are concerned with those flows in which either the angle of attack or the loading is prescribed on the wing and it is required to find solutions for the disturbance parameters in the flow field. The prescribed conditions therefore involve, in general, a specification of  $w$  or  $u$  velocity distribution on the wing. This type of problem may, in principle, be solved by use of correspondence flows in which the flow field due to arbitrarily prescribed  $w$  or  $u$  distributions is determined in terms of the flow field due to uniformly prescribed  $w$  or  $u$  distributions. Thus, in general, a prescribed  $w$  or  $u$  distribution on the wing can be expressed as a continuous function of  $x$  and  $y$  in the following form:

$$q = q_0 \sum_{n,k=0}^{n,k=\infty} \frac{a_{n,k} x^n y^k}{n! k!} \quad (\text{on wing}) \quad (3a)$$

where  $q$  represents either  $w$  or  $u$ . In equation (3a),  $q_0$  and  $a_{n,k}$  are constants. The term  $q_0$  represents a uniform prescribed velocity distribution on the wing; whereas the remaining terms represent prescribed variable chordwise or spanwise velocity distributions. Throughout the subsequent discussion, the subscript 0, when used with the general disturbance parameter  $\Phi(x,y,z)$ , refers to the flow field due to a uniform velocity on the wing. Similarly, the subscripts  $n,k$  for  $\Phi(x,y,z)$ , where  $n,k = 0,1,2,\dots,\infty$ , refer to the flow field resulting from a value of  $q$  on the wing given by

$$q_{n,k} = \frac{a_{n,k} q_0 x^n y^k}{n! k!} \quad (3b)$$

For flows in which  $k=0$ , the subscript  $k$  is omitted for brevity.

The flow field due to an arbitrary prescribed velocity on the wing may be obtained by superposing the flow fields due to an appropriate number of terms of the series in equation (3b). In the subsequent development, it is therefore sufficient to consider only the general term  $q_{n,k}$  as given by equation (3b).

#### Basic Formula

In order to simplify the subsequent derivation, the analysis is made specifically for wings with symmetrical profiles at zero lift and for lifting profiles of infinitesimal thickness. The results, however, also apply to wings of arbitrary profiles by means of superposition principles.

The boundary-value problems considered in the analysis are conveniently classified into two types. The first class is designated type A and refers to the specification of  $w$  on the wing at zero lift or of  $u$  on the lifting wing. The second class is the inverse problem, which is designated type B and refers to the specification of  $u$  on the wing at zero lift or of  $w$  on the lifting wing.

If  $w$  or  $u$  is prescribed on the wing, equation (2) for  $\Omega = q$  may be written as

$$q(x,y,z) = -K \iint_{\tau} q \frac{\partial}{\partial z} \left( \frac{1}{r} \right) d\xi d\eta \quad (4)$$

In type A boundary-value problems,  $\tau$  represents only the region included by one surface of the wing plan form; whereas in type B problems,  $\tau$  includes the entire region of disturbance over one surface of the  $z=0$  plane.

In equation (4), any of the disturbance parameters or their derivatives with respect to either  $x$  or  $y$  may be substituted for  $q$  in conjunction with the appropriate region  $\tau$  and subject to the procedure previously indicated for equation (2).

#### Correspondence Formulas for Infinite Wing

Flows related through  $x$  differentiation. - In equation (3b), let  $n \geq 1$ ; then differentiation with respect to  $x$  yields

$$(q_{n,k})_x = \frac{a_{n,k}}{a_{n-1,k}} q_{n-1,k} \quad (\text{on wing}) \quad (5a)$$



where  $q$  represents either  $w$  or  $u$  and the wing may be either at zero lift or lifting. The form expressed by equation (5a) is hereinafter referred to as "a relation through  $x$  differentiation".

When  $q$  in equation (4) is replaced first by  $q_{n-1,k}$  and then by  $(q_{n,k})_x$ , there results for the infinite wing

$$(q_{n,k})_x(x,y,z) = \frac{a_{n,k}}{a_{n-1,k}} q_{n-1,k}(x,y,z) \quad (5b)$$

Inasmuch as  $\varphi_{n-1}$  and  $\varphi_n$  vanish at  $I$ , integration of equation (5b) to obtain  $\varphi$  and differentiation of  $\varphi$  to obtain the perturbation velocities yield

$$(\Phi_{n,k})_x(x,y,z) = \frac{a_{n,k}}{a_{n-1,k}} \Phi_{n-1,k}(x,y,z) \quad (5c)$$

or, at the point  $x,y,z$ ,

$$\Phi_{n,k} = \frac{a_{n,k}}{a_{n-1,k}} \int_I^x \Phi_{n-1,k} d\xi \quad (5d)$$

where  $\Phi$  represents any of the disturbance parameters  $\varphi$ ,  $u$ ,  $v$ , or  $w$ .

Equation (5d) has the following significance:

If an infinite wing at either subsonic or supersonic speed is given in which the wing profile is either symmetrical and at zero lift or infinitesimally thick and lifting and with either  $w$  or  $u$  prescribed on the wing according to equation (3b), then each of the parameters  $\varphi$ ,  $u$ ,  $v$ , or  $w$  for the  $n,k$  flow is related to the corresponding parameter for the  $n-1,k$  flow according to equation (5d). Furthermore, by integration of equation (5d) for  $\Phi = u$ , there results at the point  $x,y,z$

$$u_{n,k} = \frac{a_{n,k}}{a_{n-1,k}} u_{n-1,k} \quad (5e)$$

Flows related through  $y$  differentiation. - In equation (3b) let  $k \geq 1$ ; then differentiation with respect to  $y$  yields

$$(q_{n,k})_y = \frac{a_{n,k}}{a_{n,k-1}} q_{n,k-1} \quad (\text{on wing}) \quad (6a)$$

The form expressed by equation (6a) is hereinafter referred to as "a relation through y differentiation".

When  $q$  in equation (4) is replaced first by  $q_{n,k-1}$  and then by  $(q_{n,k})_y$ , there results for the infinite wing

$$(q_{n,k})_y(x,y,z) = \frac{a_{n,k}}{a_{n,k-1}} q_{n,k-1}(x,y,z) \quad (6b)$$

and by the same reasoning that was previously indicated for the flow  $(q_{n,k})_x$ , there results

$$(\Phi_{n,k})_y(x,y,z) = \frac{a_{n,k}}{a_{n,k-1}} \Phi_{n,k-1}(x,y,z) \quad (6c)$$

or, at the point  $x,y,z$ ,

$$\Phi_{n,k} = \frac{a_{n,k}}{a_{n,k-1}} \int_{I,0}^y \Phi_{n,k-1} d\eta \quad (6d)$$

where, as before,  $\Phi$  represents any of the disturbance parameters  $\phi$ ,  $u$ ,  $v$ , or  $w$ . The two lower limits  $I$  and  $0$  in equation (6d) refer to the conditions that  $k$  is even or odd, respectively. If  $k$  is odd,  $\Phi_{n,k}$  vanishes in the  $y=0$  plane and  $I$  in equation (6d) may be replaced by  $0$ . By integration of equation (6d) for  $\Phi = v$ , there results at the point  $x,y,z$

$$v_{n,k} = \frac{a_{n,k}}{a_{n,k-1}} \phi_{n,k-1} \quad (6e)$$

Flows related through arbitrary differentiation. - In the preceding cases, the differentiation for  $q$  on the wing was performed with respect to either  $x$  or  $y$ . Other extensions leading to results corresponding to equations (5) and (6) may be obtained by a similar procedure through any number of successive differentiations with

respect to either  $x$  alone,  $y$  alone, or both  $x$  and  $y$ . In this manner, the flow field due to an arbitrary prescribed velocity on the wing may be expressed in terms of the flow field due to a uniform velocity on the wing.

## METHOD OF EDGE CORRECTIONS

### General Procedure

The previous section treated infinite wings, wherein the prescribed relations on the wing given by equations (5a) and (6a) are valid over the entire plan form. In a finite wing of arbitrary plan form, however, the wing boundary may alter these prescribed relations on the edges in order to conform with conditions (2) given in the section Basic Theory. Consequently, the application of equations (5) and (6) to finite plan forms requires the superposition of expressions that correct for the edges.

The edge corrections involve the cancellation in the plane of the wing of magnitudes of  $u$  or  $w$ , as determined from equations (5d), (5e), (6d), (6e), and conditions (2), without disturbing the prescribed  $q$  on the wing. If the prescribed flow on the wing is related through  $x$  differentiation (equation (5a)), corrections may be required only for the leading and trailing edges; whereas if the prescribed flow on the wing is related through  $y$  differentiation (equation (6a)), corrections may be required only for the side edges. A general procedure for canceling  $q$  off the wing is the utilization of equation (4) in conjunction with the addition of a fictitious wing placed in the  $z=0$  plane. This procedure offers no complications in type A boundary-value problems, but may involve the solution of an integral equation in type B boundary-value problems. In specific problems, however, other methods for canceling  $q$  off the wing may be found more convenient.

### Type A Boundary-Value Problem

If  $w$  is prescribed on the wing of zero lift or if  $u$  is prescribed on the lifting wing, the edge corrections can be determined by use of equation (4). The procedure is to substitute for  $q$  in the integrand of equation (4) the required change in this parameter evaluated across the edge. In this operation, only those edges that alter the given prescribed relation on the wing, expressed in derivative form, require consideration. (See the appendix for illustration.)

Flows related through  $x$  differentiation. - If the prescribed flow on the wing is related through  $x$  differentiation, an edge

correction is required only for the trailing edge in order to satisfy the boundary conditions in the wake of the wing. The trailing-edge correction is required to cancel in the wing wake a magnitude of  $w$  in the zero-lift condition and a magnitude of  $u$  in the lifting condition. The fact that, in this type of flow, only a trailing-edge correction is required is significant at supersonic speeds in that the infinite-wing relations apply to finite plan forms at all points outside of the Mach waves from the trailing edge. The infinite-wing relations also apply at subsonic and supersonic speeds if the wing chord extends backward to infinity, even though the span is finite.

Flows related through  $y$  differentiation. - If the prescribed flow on the wing is related through  $y$  differentiation, corrections are required only for those edges that are inclined to the  $y$ -axis; that is, side edges. The side-edge corrections are required to cancel outboard of the side edges a magnitude of  $w$  in the zero-lift condition and a magnitude of  $u$  in the lifting condition. The fact that, in this type of flow relation, only side-edge corrections are required is significant at supersonic speeds in that the infinite-wing relations apply at all points outside of the Mach waves from the side edges. The infinite-wing relations also apply at subsonic and supersonic speeds if the side edges are at infinity, even though the chord is finite (rectangular wing of infinite span).

#### Type B Boundary-Value Problem

In general, the method for determining the edge corrections for the inverse problem in which  $u$  is prescribed on the wing of zero lift or  $w$  is prescribed on the lifting wing is more complicated than for the type A problem. In solving the inverse problem, difficulty arises because the specification of  $u$  only on the zero-lift wing leaves  $u$  undetermined in the remainder of the plane of the wing. Similarly, the specification of  $w$  only on the lifting wing leaves  $w$  undetermined in the remainder of the plane of the wing. In the type B problem, the distribution function for the singularities is therefore incompletely specified in the integrand of equation (2). In some cases, this problem may be solved without requiring the solution of an integral equation.

Flows related through  $x$  differentiation. - If the prescribed flow on the wing is related through  $x$  differentiation, corrections are required only for the subsonic leading edge and for the trailing edge at all speeds. The subsonic-leading-edge correction is required to cancel a magnitude of  $q$  on the wing. The trailing-edge correction is required to cancel in the wing wake a magnitude of  $w$  in the zero-lift condition and a magnitude of  $u$  in the lifting condition

without disturbing the prescribed values of  $q$  on the wing. If the leading edge is supersonic, an edge correction for this type of flow relation is required only for the trailing edge and the infinite-wing relations apply to the finite plan form at all points outside of the Mach cones from the trailing edge. If the trailing edge is supersonic, the required cancellation of  $w$  or  $u$  in the wing wake is accomplished without requiring the solution of an integral equation by placing a fictitious wing in the wake and then applying equation (4).

Flows related through  $y$  differentiation. - If the prescribed flow on the wing is related through  $y$  differentiation, the required edge corrections can be determined without solving an integral equation if the side edges are supersonic. For this type of flow relation, the infinite-wing relations apply to the same cases previously indicated for type A problems for the corresponding prescribed flow relation.

#### CORRESPONDENCE FORMULAS FOR RECTANGULAR WINGS

##### Type A Boundary-Value Problem

In the following analysis, let  $q$  in equation (3b) represent either a prescribed  $w$  for the wing of zero lift or a prescribed  $u$  for the lifting wing.

Flows related through  $x$  differentiation. - For flows related through  $x$  differentiation, consider a rectangular wing on which the prescribed velocity is given by equation (3b); then the relation

$$(q_{n,k})_x = \frac{a_{n,k}}{a_{n-1,k}} q_{n-1,k} \quad (\text{on wing}) \quad (7a)$$

identically applies on the leading and side edges. Across the trailing edge, however, condition 2(a) or 2(b), presented in the section Basic Theory, requires a decrease in  $q_{n,k}$ ,

$$(\Delta q_{n,k})_T = - \frac{a_{n,k} q_0 c^n y^k}{n! k!} \quad (7b)$$

where  $c$  is the chord. Consequently, the evaluation of  $(q_{n,k})_x(x,y,z)$  from equation (4) ( $(q_{n,k})_x$  is substituted for  $q$ ) requires a correction only for the trailing edge. The details of the evaluation of the trailing-edge correction are given in the appendix. The result at a point  $x,y,z$  is

$$\Delta\phi_{n,k} = \frac{a_{n,k} c^n}{a_{0,k} n!} (\Delta\phi_{0,k})_T \quad (A6)$$

where  $(\Delta\phi_{0,k})_T$  refers to the effect of the wing cut-off at the trailing edge; that is, the effect of the change in  $q_{0,k}$  along the trailing edge. (See the appendix.) In the  $z=0$  plane, in the zero-lift case for  $\phi=w$  and in the lifting case for  $\phi=u$ , the quantity  $(\Delta\phi_{0,k})_T$  is zero everywhere except in the wake, where the value is  $-q_{0,k}$ . The application of equation (A6) may be extended to plan forms other than the rectangular in the sense that it represents the trailing-edge correction for any plan form with a straight trailing edge.

The correspondence relation expressed by equation (A6) represents the edge corrections for finite rectangular wings on which the prescribed velocity is related through  $x$  differentiation. The complete correspondence relations for this flow and plan form are obtained by adding equation (A6) to equation (5d) or (5e). Because rectangular wings for this condition require a correction only for the trailing edge, it follows that the correspondence relations (5d) and (5e) are, without further correction, applicable to the following cases:

- (a) Subsonic flow for an arbitrary span length and a chord extending backward to infinity
- (b) Supersonic flow for any rectangular plan form at all points outside of the trailing-edge Mach cone

Flows related through  $y$  differentiation. - For flows related through  $y$  differentiation, let equation (3b) be differentiated with respect to  $y$ ; then the relation

$$(q_{n,k})_y = \frac{a_{n,k}}{a_{n,k-1}} q_{n,k-1} \quad (\text{on wing}) \quad (8a)$$

applies identically on the leading and trailing edges. Across each side edge, however, condition 2(a) or 2(b), presented in the section Basic Theory, requires a change in  $q_{n,k}$

$$\Delta q_{n,k} = - \frac{a_{n,k} q_0 (\pm h)^k x^n}{n! k!} \quad (A9)$$

where  $h$  is the semispan and the plus and minus signs preceding  $h$  refer to the right and left side edges, respectively. The evaluation of  $(q_{n,k})_y$  from equation (4) ( $(q_{n,k})_y$  is substituted for  $q$ ) therefore requires a correction only for the side edges. The details of the evaluation of the side-edge correction are given in the appendix. The result at a point  $x, y, z$  is

$$\Delta \Phi_{n,k} = \frac{a_{n,k}}{a_n k!} \left[ h^k (\Delta \Phi_n)_{RS} + (-h)^k (\Delta \Phi_n)_{LS} \right] \quad (A15)$$

where  $(\Delta \Phi_n)_{RS}$  and  $(\Delta \Phi_n)_{LS}$  refer to the effect of the wing cut-off along the side edges; that is, the effect of the change in  $q_n$  along the side edges. In the  $z=0$  plane, in the zero-lift case for  $\Phi=w$  and in the lifting case for  $\Phi=u$ , the quantities  $(\Delta \Phi_n)_{RS}$  and  $(\Delta \Phi_n)_{LS}$  are zero everywhere except outboard of the side edges, where the value is  $-q_n$ . The application of equation (A15) may be extended to plan forms other than the rectangular in the sense that equation (A15) represents the side-edge correction for any plan form with side edges in the streamwise direction.

The correspondence relation expressed by equation (A15) represents the edge corrections for finite rectangular wings on which the prescribed  $q$  is related through  $y$  differentiation. The complete correspondence relations for this condition are obtained by adding equation (A15) to equation (6d) or (6e). Because rectangular wings for this condition require an edge correction only for the side edges, it follows that the correspondence equations (6d) and (6e) are applicable, without further corrections, to the following cases:

- (a) Subsonic flow for an infinite span and an arbitrary chord length
- (b) Supersonic flow for any rectangular plan form at all points outside of the Mach aft cones from the side edges

#### Type B Boundary-Value Problem

In the following analysis, let  $q$  in equation (3b) represent either a prescribed  $u$  for the wing of zero lift or a prescribed  $w$

for the lifting wing. The subsequent treatment for the rectangular wing considers the type B problem in cases where the solution of an integral equation is not required.

Flows related through  $x$  differentiation. - If the flow is related on the wing through  $x$  differentiation, corrections for both the leading and trailing edges are required at subsonic speeds. At supersonic speeds, however, only a trailing-edge correction is required. Thus, equations (5d) and (5e) apply to the rectangular wing at supersonic speeds for all points outside of the Mach waves from the trailing edge.

In the Ackeret region of the wing, where  $u$  is proportional to  $w$ , the type B problem is transformed into a type A problem so that the trailing-edge correction expressed by equation (A6) is also applicable to points in the flow field influenced only by the Ackeret region. For points in the flow field within the Mach cone from the trailing edge and influenced only by the Ackeret region, the correction expressed by equation (A6) is added to equation (5d) or (5e).

The trailing-edge correction at supersonic speeds for points influenced by the side edge can be determined by superposing a fictitious wing in the wake such that

$$\Delta u_{n,k}(x,y,0) = - \frac{a_{n,k}}{a_{n-1,k}} \varphi_{n-1,k}(x,y,0) \quad (9a)$$

By utilizing equation (4), with  $q$  replaced by  $\Delta u_{n,k}$ , there results

$$\begin{aligned} \Delta u_{n,k}(x,y,z) &= K \frac{a_{n,k}}{a_{n-1,k}} \iint_{\text{wake}} \varphi_{n-1,k} \frac{\partial}{\partial z} \left( \frac{1}{r} \right) d\xi d\eta \\ &= - \frac{a_{n,k}}{a_{n-1,k}} (\Delta \varphi_{n-1,k})_{\text{wake}}(x,y,z) \end{aligned} \quad (9b)$$

where  $(\Delta \varphi_{n-1,k})_{\text{wake}}$  is the contribution of the wake to the velocity potential in the  $n-1,k$  flow and  $\Delta u_{n,k}$  is the entire edge correction for  $u_{n,k}$ . Integration of equation (9b) to obtain  $\Delta \varphi_{n,k}$  and differentiation to obtain the perturbation velocities yield



$$\Delta\phi_{n,k}(x,y,z) = - \frac{a_{n,k}}{a_{n-1,k}} \int_{I_T}^x (\Delta\phi_{n-1,k})_{\text{wake}} d\xi \quad (9c)$$

where the limit  $I_T$  indicates that the flow is initiated from the trailing edge and  $(\Delta\phi_{n-1,k})_{\text{wake}}$  refers to the contribution of the wake to  $\phi$ ,  $u$ ,  $v$ , or  $w$ . Equations (9b) and (9c), when applied to points within the trailing-edge Mach cone that are influenced only by the Ackeret region, represent additional forms of the trailing-edge correction other than that given previously for this region.

A summary of the correspondence formulas for the rectangular wing is presented in table I.

#### EXAMPLES OF USE OF CORRESPONDENCE FLOWS

A few examples of the use of correspondence flows for the type B problem are given for the lifting rectangular wing at supersonic speeds. Let  $q_{n,k}$  assume the following prescribed velocities on the wing:

$$w_1 = a_1 w_0 x \quad (10a)$$

$$w_{0,1} = a_{0,1} w_0 y \quad (10b)$$

$$w_{1,1} = a_{1,1} w_0 xy \quad (10c)$$

The disturbance velocity  $u$  for the flows represented by equations (10) are obtained in terms of the solutions for the wing at a constant angle of attack for which  $w_0 = -V\alpha$ . The lettered regions in the subsequent discussion refer to figure 1.

$$\text{Example A: } w_1 = a_1 w_0 x \quad (\text{on wing})$$

The flow expressed by equation (10a) represents a wing pitching steadily at an angular velocity  $q' = -a_1 w_0$ .

Region A. - In region A between the side-edge Mach cones and outside of the trailing-edge Mach waves, the potential function for a uniform angle of attack  $\alpha$  is

$$\varphi_0(x,y,z) = -\frac{w_0}{B} (x-Bz) \quad (11a)$$

Then, from equation (5e) ( $n=1, k=0, a_0=1$ ),

$$\begin{aligned} u_1(x,y,z) &= a_1 \varphi_0(x,y,z) \\ &= \frac{q'}{B} (x-Bz) \end{aligned} \quad (11b)$$

Region B. - In region B between the side-edge Mach cones and within the trailing-edge Mach waves, the potential function for a uniform angle of attack  $\alpha$  is given by

$$\varphi_0(x,y,z) = -\frac{w_0^c}{B} \quad (12a)$$

In this region, equations (5e) and (A6) are applicable and

$$u_1(x,y,z) = a_1 \varphi_0 + a_1 c(\Delta u_0)_T \quad (12b)$$

where, by use of equations (11a) and (12a),  $(\Delta u_0)_T = \frac{w_0}{B}$ . Therefore,

$$u_1(x,y,z) = \frac{q'}{B} - \frac{q'}{B} = 0 \quad (12c)$$

Region C. - In region C within the Mach cone from one side edge and outside of the trailing-edge Mach waves, the potential function  $\varphi_0$  in the  $z=0$  plane is given by (reference 4, table I)

$$\varphi_0(x,y,0) = -\frac{w_0}{\pi} \left[ \frac{x}{B} \cos^{-1} \left( \frac{2y_a B}{x} + 1 \right) + 2 \sqrt{-y_a \left( y_a + \frac{x}{B} \right)} \right] \quad (13a)$$

where  $y_a$  refers to an origin at the leading edge of the tip section; that is,

$$y_a = y - h \quad (\text{on right half-wing})$$

In this region, equation (5e) is applicable in accordance with correspondence rules for flows related through  $x$  differentiation at supersonic speeds. Therefore,

$$\begin{aligned} u_1(x, y, 0) &= a_1 \varphi_0(x, y, 0) \\ &= \frac{q'}{\pi} \left[ \frac{x}{B} \cos^{-1} \left( \frac{2y_a B}{x} + 1 \right) + 2 \sqrt{-y_a \left( y_a + \frac{x}{B} \right)} \right] \end{aligned} \quad (13b)$$

(Equation (13b) agrees with the result obtained in table I of reference 4 when the pitching axis in reference 4 is referred to the leading edge.)

Region D. - In region D within the Mach cone from one side edge and also within the trailing-edge Mach waves, equation (9b) is applicable. For points in the  $z=0$  plane, a doublet singularity has no effect except in the immediate vicinity of its position; therefore,

$$\Delta u_1(x, y, 0) = -a_1 \varphi_0(x, y, 0) \quad (14a)$$

and

$$u_1(x, y, 0) = a_1 \varphi_0(x, y, 0) - a_1 \varphi_0(x, y, 0) = 0 \quad (14b)$$

$$\text{Example B: } w_{0,1} = a_{0,1} w_0 \quad (\text{on wing})$$

The flow expressed by equation (10b) represents a wing rolling steadily at an angular velocity  $p = a_{0,1} w_0$ .

Region A. - In region A between the side-edge Mach cones and outside of the trailing-edge Mach waves, the disturbance velocity  $u_0$  for a uniform angle of attack  $\alpha$  is

$$u_0(x, y, z) = -\frac{w_0}{B}$$

In this region, equation (6d) is applicable ( $k=1$ ,  $a_0=1$ ,  $\Phi=u$ ) and

$$\begin{aligned} u_{0,1}(x,y,z) &= a_{0,1} \int_0^y u_0 \, d\eta \\ &= \frac{py}{B} \end{aligned} \quad (15)$$

Region B. - In region B between the side-edge Mach cones and within the trailing-edge Mach waves, the disturbance velocity  $u_0(x,y,z) = 0$ . In this region, equation (6d) is still applicable and

$$\begin{aligned} u_{0,1}(x,y,z) &= a_{0,1} \int_0^y u_0 \, d\eta \\ &= 0 \end{aligned} \quad (16)$$

Example C:  $w_{1,1} = a_{1,1} w_{0,xy}$  (on wing)

Region A. - If  $w_{1,1}$  in equation (10c) is differentiated with respect to  $x$ , then in region A between the side-edge Mach cones and outside of the trailing-edge Mach waves, equation (5e) is applicable ( $n=1$ ,  $k=1$ ) and

$$u_{1,1}(x,y,z) = \frac{a_{1,1}}{a_{0,1}} \varphi_{0,1}(x,y,z)$$

where  $\varphi_{0,1}$  is obtained from equation (6d) ( $\Phi=\varphi$ ,  $a_0=1$ ) as

$$\varphi_{0,1}(x,y,z) = a_{0,1} \int_0^y \varphi_0 \, d\eta$$

where  $\varphi_0$  is given by equation (11a). Then

$$\begin{aligned}\varphi_{0,1}(x,y,z) &= -\frac{a_{0,1}w_0}{B}(x-Bz) \int_0^y d\eta \\ &= -\frac{a_{0,1}w_0y}{B}(x-Bz)\end{aligned}$$

Therefore

$$u_{1,1}(x,y,z) = -\frac{a_{1,1}w_0y}{B}(x-Bz) \quad (17)$$

Region B. - In region B between the side-edge Mach cones and within the trailing-edge Mach waves, equations (5e) and (A6) are applicable and

$$u_{1,1}(x,y,z) = a_1\varphi_{0,1}(x,y,z) + a_1c(\Delta u_{0,1})_T \quad (18a)$$

where  $\varphi_{0,1}$  is obtained from equation (6d) ( $\phi=\varphi$ ). Thus

$$\varphi_{0,1}(x,y,z) = a_{0,1} \int_0^y \varphi_0 d\eta$$

where, from equation (12a),

$$\varphi_0(x,y,z) = -\frac{w_0c}{B}$$

Then

$$\varphi_{0,1}(x,y,z) = -\frac{a_{0,1}w_0cy}{B} \quad (18b)$$

From equations (15) and (16),  $(\Delta u_{0,1})_T$  in equation (18a) has the value  $-py/B$ . By substituting for  $\varphi_{0,1}$  and for  $(\Delta u_{0,1})_T$ , there results

$$\begin{aligned}u_{1,1}(x,y,z) &= -\frac{a_{1,1}w_0cy}{B} + \frac{a_{1,1}w_0cy}{B} \\ &= 0\end{aligned} \quad (18c)$$

## CONCLUDING REMARKS

The method of relating the solution of a flow field to the solution of a simpler or known boundary-value problem has been extended, subject to edge corrections for finite plan forms, to the most general type of prescribed boundary conditions on a wing in the linearized potential field. By means of this procedure and within these limitations, the flow field due to an arbitrary prescribed velocity distribution on the wing can be expressed in terms of the flow field due to a uniform prescribed velocity on the wing.

A generalized correspondence rule has been demonstrated for wings in linearized potential flow for subsonic and supersonic speeds. This rule states:

1. If any of the derivatives of a prescribed velocity distribution is related on the wing by a constant to another prescribed distribution of the velocity or to any of its derivatives, the given relation is valid, subject to edge corrections, throughout the entire flow field for any corresponding disturbance parameter of the two flow fields.

2. If the prescribed flow on the wing is related through differentiation in the stream direction (with respect to  $x$ ), corrections may be required only for the leading and trailing edges; whereas if the prescribed flow on the wing is related through differentiation in the lateral direction (with respect to  $y$ ), corrections may be required only for the side edges.

Lewis Flight Propulsion Laboratory,  
National Advisory Committee for Aeronautics,  
Cleveland, Ohio, November 3, 1950.

## APPENDIX

DERIVATION OF EDGE CORRECTIONS FOR  
FINITE RECTANGULAR WINGS

The derivation for the edge corrections is applicable to finite rectangular wings at subsonic and supersonic speeds for symmetrical profiles at zero lift on which  $w$  is prescribed and for lifting profiles of infinitesimal thickness on which  $u$  is prescribed.

Flows Related Through  $x$  Differentiation

Consider the flow

$$q_{n,k} = \frac{a_{n,k} q_0 x^n y^k}{n! k!} \quad (\text{on wing}) \quad (3b)$$

$$n = 1, 2, \dots, \infty; k = 0, 1, 2, \dots, \infty$$

Then the relation

$$(q_{n,k})_x = \frac{a_{n,k}}{a_{n-1,k}} q_{n-1,k} \quad (\text{on wing}) \quad (5a)$$

applies identically on the leading and side edges. Across the trailing edge, however, condition 2(a) or 2(b), presented in the section Basic Theory, requires a decrease in  $q_{n,k}$ ,

$$(\Delta q_{n,k})_T = - \frac{a_{n,k} q_0 c^n y^k}{n! k!} \quad (7b)$$

where  $c$  is the chord.

The change in  $q_{n,k}$  is assumed to occur across a strip of width  $\Delta \xi$  on the trailing edge. This change contributes an increment in  $(q_{n,k})_x(x,y,z)$ , which is the entire edge correction and, according to equation (4) with the appropriate substitutions for  $q$ , is

$$\Delta(q_{n,k})_x(x,y,z) = -K \int_T \frac{(\Delta q_{n,k})_T}{\Delta \xi} \frac{\partial}{\partial z} \left( \frac{1}{r} \right) \Delta \xi \, d\eta \quad (A1)$$

where the integral is evaluated along the trailing edge and  $(\Delta q_{n,k})_T$  is given by equation (7b).

In a similar manner,

$$[\Delta(q_{0,k})_x]_T(x,y,z) = -K \int_T \frac{(\Delta q_{0,k})_T}{\Delta \xi} \frac{\partial}{\partial z} \left( \frac{1}{r} \right) \Delta \xi \, d\eta \quad (A2)$$

where  $[\Delta(q_{0,k})_x]_T$  refers to the contribution of the trailing edge to  $(q_{0,k})_x$  and

$$(\Delta q_{0,k})_T = - \frac{a_{0,k} q_0 y^k}{k!} \quad (A3)$$

When the integrals of equations (A1) and (A2) are compared, it is seen that

$$\Delta(q_{n,k})_x(x,y,z) = \frac{a_{n,k} c^n}{a_{0,k} n!} [\Delta(q_{0,k})_x]_T(x,y,z) \quad (A4)$$

Integration of equation (A4) yields, at the point  $x,y,z$ ,

$$\Delta q_{n,k} = \frac{a_{n,k} c^n}{a_{0,k} n!} (\Delta q_{0,k})_T \quad (A5)$$

Inasmuch as  $\varphi_{n,k}$  and  $\varphi_{0,k}$  must vanish at the lower limit (See boundary condition (3) given in section Basic Theory.), integration of equation (A5) to obtain  $\varphi$  and differentiation of  $\varphi$  to obtain the perturbation velocities yield, at the point  $x,y,z$ ,

$$\Delta \Phi_{n,k} = \frac{a_{n,k} c^n}{a_{0,k} n!} (\Delta \Phi_{0,k})_T \quad (A6)$$



The term  $(\Delta\phi_{0,k})_T$  refers to the effect of the wing cut-off at the trailing edge; that is, the effect of the change in  $q_{0,k}$  along the trailing edge. This term can, in general, be determined by use of equation (A2). In specific cases, however, it may be simpler to use various other modifications of equation (2). Thus, in the zero-lift case, equation (2) for  $\Omega = u$  and  $u_z = w_x$  yields

$$(\Delta u_{0,k})_T(x,y,z) = -K \int_T \frac{(\Delta w_{0,k})_T}{\Delta \xi} \frac{1}{r} \Delta \xi \, d\eta \quad (A7)$$

where  $(\Delta w_{0,k})_T$  is given by equation (A3) and

$$(\Delta u_{0,k})_T(x,y,z) = -K \frac{a_{0,k} q_0}{k!} \int_T \frac{\eta^k}{r} \, d\eta \quad (A8)$$

#### Flows Related Through $y$ Differentiation

Consider the flow

$$q_{n,k} = \frac{a_{n,k} q_0 x^n y^k}{n! k!} \quad (\text{on wing}) \quad (3b)$$

$$k = 1, 2, \dots, \infty; \quad n = 0, 1, 2, \dots, \infty$$

Then the relation

$$(q_{n,k})_y = \frac{a_{n,k}}{a_{n,k-1}} q_{n,k-1} \quad (\text{on wing}) \quad (6a)$$

applies identically along the leading and trailing edges. Across a side edge, however, condition 2(a) or 2(b), presented in the section Basic Theory, require a change in  $q_{n,k}$

$$\Delta q_{n,k} = - \frac{a_{n,k} q_0 (\pm h)^k x^n}{n! k!} \quad (A9)$$

where  $h$  is the semispan and the plus and minus sign preceding  $h$  refers to the right and left side edges, respectively.

This change in  $q_{n,k}$  is assumed to occur along a strip of width  $\Delta\eta$  on each side edge and contributes an increment in  $(q_{n,k})_y(x,y,z)$ , which is the entire edge correction and, according to equation (4) with the appropriate substitutions for  $q$ , is

$$\Delta(q_{n,k})_y(x,y,z) = -K \int_{RS} \frac{(\Delta q_{n,k})_{RS}}{\Delta\eta} \frac{\partial}{\partial z} \left( \frac{1}{r} \right) \Delta\eta d\xi - K \int_{LS} \frac{(\Delta q_{n,k})_{LS}}{\Delta\eta} \frac{\partial}{\partial z} \left( \frac{1}{r} \right) \Delta\eta d\xi \quad (A10)$$

where the integral is evaluated along each side edge and  $(\Delta q_{n,k})$  for each side edge is given by equation (A9).

In a similar manner,

$$\left[ \Delta(q_n)_y \right]_S(x,y,z) = -K \int_{RS} \frac{(\Delta q_n)_{RS}}{\Delta\eta} \frac{\partial}{\partial z} \left( \frac{1}{r} \right) \Delta\eta d\xi - K \int_{LS} \frac{(\Delta q_n)_{LS}}{\Delta\eta} \frac{\partial}{\partial z} \left( \frac{1}{r} \right) \Delta\eta d\xi \quad (A11)$$

where  $\left[ \Delta(q_n)_y \right]_S$  refers to the contribution of the side edges to  $(q_n)_y$  and where, for each side edge,

$$(\Delta q_n)_S = - \frac{a_n q_0 x^n}{n!} \quad (A12)$$

When the integrals of equations (A10) and (A11) are compared, it is seen that

$$\Delta(q_{n,k})_y(x,y,z) = \frac{a_{n,k}}{a_n k!} \left\{ h^k \left[ \Delta(q_n)_y \right]_{RS} + (-h)^k \left[ \Delta(q_n)_y \right]_{LS} \right\} \quad (A13)$$

Integration of equation (A13) yields, at the point  $x, y, z$ ,

$$\Delta q_{n,k} = \frac{a_{n,k}}{a_n k!} \left[ h^k (\Delta q_n)_{RS} + (-h)^k (\Delta q_n)_{LS} \right] \quad (A14)$$

Inasmuch as  $\varphi_{n,k}$  and  $\varphi_n$  must vanish at the lower limit, integration of equation (A14) to obtain  $\varphi$  and differentiation of  $\varphi$  to obtain the perturbation velocities yield, at the point  $x, y, z$ ,

$$\Delta \Phi_{n,k} = \frac{a_{n,k}}{a_n k!} \left[ h^k (\Delta \Phi_n)_{RS} + (-h)^k (\Delta \Phi_n)_{LS} \right] \quad (A15)$$

The terms  $(\Delta \Phi_n)_{RS}$  and  $(\Delta \Phi_n)_{LS}$  refer to the effect of the wing cut-off along the side edges; that is, the effect of the change in  $q_n$  along the side edges. These terms can, in general, be determined by use of equation (A11). In specific cases, however, it may be simpler to use various other modifications of equation (2). Thus, in the zero-lift case, equation (2) for  $\Omega = v$  and  $v_z = w_y$  yields

$$(\Delta v_n)_{RS}(x, y, z) = -K \int_{RS} \frac{(\Delta w_n)_{RS}}{\Delta \eta} \frac{1}{r} \Delta \eta \, d\xi$$

(where  $(\Delta w_n)_{RS}$  is given by equation (A12)) and similarly for  $(\Delta v_n)_{LS}$ . Thus,

$$(\Delta v_n)_{RS}(x, y, z) = -K \frac{a_n^{q_0}}{n!} \int_{RS} \frac{\xi^n}{r} \, d\xi \quad (A16a)$$

$$(\Delta v_n)_{LS}(x, y, z) = -K \frac{a_n^{q_0}}{n!} \int_{LS} \frac{\xi^n}{r} \, d\xi \quad (A16b)$$

## REFERENCES

1. Lamb, Horace: Hydrodynamics. Cambridge Univ. Press, 6th ed., 1932, pp. 58-61.
2. Heaslet, Max A., and Lomax, Harvard: The Use of Source-Sink and Doublet Distributions Extended to the Solution of Arbitrary Boundary-Value Problems in Supersonic Flow. NACA Rep. 900, 1948. (Formerly NACA TN 1515.)
3. Lagerstrom, P. A., and Graham, Martha E.: Downwash and Sidewash Induced by Three-Dimensional Lifting Wings in Supersonic Flow. Rep. No. SM-13007, Douglas Aircraft Co., Inc., April 1947.
4. Harmon, Sidney M.: Stability Derivatives at Supersonic Speeds of Thin Rectangular Wings with Diagonals Ahead of Tip Mach Lines. NACA Rep. 925, 1949. (Formerly NACA TN 1706.)

TABLE I - SUMMARY OF CORRESPONDENCE FORMULAS FOR RECTANGULAR WINGS

| Prescribed velocity on wing                                                                              | Plan-form configuration                                                               | Speed                | Type of boundary-value problem (a) | Formula at x, y, z                                                                                                                                                                                            | NACA |
|----------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|----------------------|------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| $q_{n,k} = \frac{a_n k Q_0 x^{n,k}}{n! k!}$<br>$n = 1, 2, \dots, \infty$<br>$k = 0, 1, 2, \dots, \infty$ | Infinite wing                                                                         | Subsonic, supersonic | A, B                               | $\phi_{n,k} = \frac{a_n k}{a_{n-1,k}} \int_I^x \phi_{n-1,k} dt$                                                                                                                                               |      |
|                                                                                                          | Finite span with chord extending to infinity                                          | Supersonic           | A, B                               |                                                                                                                                                                                                               |      |
|                                                                                                          | All points outside trailing-edge Mach waves; any wing                                 | Supersonic           | A, B                               | $u_{n,k} = \frac{a_n k}{a_{n-1,k}} \phi_{n-1,k}$                                                                                                                                                              |      |
|                                                                                                          | Finite wing                                                                           | Subsonic, supersonic | A                                  |                                                                                                                                                                                                               |      |
| $n = 1, 2, \dots, \infty$<br>$k = 0, 1, 2, \dots, \infty$                                                | All points within trailing-edge Mach waves but outside side-edge Mach waves; any wing | Supersonic           | B (also A)                         | $\phi_{n,k} = \frac{a_n k}{a_{n-1,k}} \int_I^x \phi_{n-1,k} dt + \frac{a_n k c^n}{a_{0,k} n!} (\Delta \phi_{0,k})_T$                                                                                          |      |
|                                                                                                          | Points within Mach aft cones from trailing and side edges; any wing                   | Supersonic           | B (also A)                         | $u_{n,k} = \frac{a_n k}{a_{n-1,k}} \phi_{n-1,k} - \frac{a_n k c^n}{a_{0,k} n!} (\Delta u_{0,k})_T$                                                                                                            |      |
|                                                                                                          |                                                                                       |                      |                                    | $\phi_{n,k} = \frac{a_n k}{a_{n-1,k}} \left[ \int_I^x \phi_{n-1,k} dt - \int_{I_T}^x (\phi_{n-1,k})_{wake} dt \right]$                                                                                        |      |
|                                                                                                          |                                                                                       |                      |                                    |                                                                                                                                                                                                               |      |
| $q_{n,k} = \frac{a_n k Q_0 x^{n,k}}{n! k!}$<br>$k = 1, 2, \dots, \infty$<br>$n = 0, 1, 2, \dots, \infty$ | Infinite wing                                                                         | Subsonic, supersonic | A, B                               | $\phi_{n,k} = \frac{a_n k}{a_{n,k-1}} \int_I^x \phi_{n,k-1} d\eta \quad (I = 0 \text{ if } k \text{ is odd})$                                                                                                 |      |
|                                                                                                          | Finite chord with infinite span                                                       | Subsonic, supersonic | A, B                               |                                                                                                                                                                                                               |      |
|                                                                                                          | All points outside side-edge Mach waves; any wing                                     | Supersonic           | A, B                               | $v_{n,k} = \frac{a_n k}{a_{n,k-1}} \phi_{n,k-1}$                                                                                                                                                              |      |
|                                                                                                          | Finite wing                                                                           | Subsonic, supersonic | A                                  |                                                                                                                                                                                                               |      |
|                                                                                                          |                                                                                       |                      |                                    | $\phi_{n,k} = \frac{a_n k}{a_{n,k-1}} \int_I^x \phi_{n,k-1} d\eta + \frac{a_n k}{a_{n,k-1}} \left[ h^k (\Delta \phi_n)_{RS} + (-h)^k (\Delta \phi_n)_{LS} \right] \quad (I = 0 \text{ if } k \text{ is odd})$ |      |
|                                                                                                          |                                                                                       |                      |                                    | $v_{n,k} = \frac{a_n k}{a_{n,k-1}} \phi_{n,k-1} + \frac{a_n k}{a_{n,k-1}} \left[ h^k (\Delta v_n)_{RS} + (-h)^k (\Delta v_n)_{LS} \right]$                                                                    |      |

<sup>a</sup>Type A refers to problem in which either  $w$  is prescribed on symmetrical profile at zero lift or  $u$  is prescribed on lifting profile of infinitesimal thickness; type B refers to problem in which either  $u$  is prescribed on symmetrical profile at zero lift or  $w$  is prescribed on lifting profile of infinitesimal thickness.



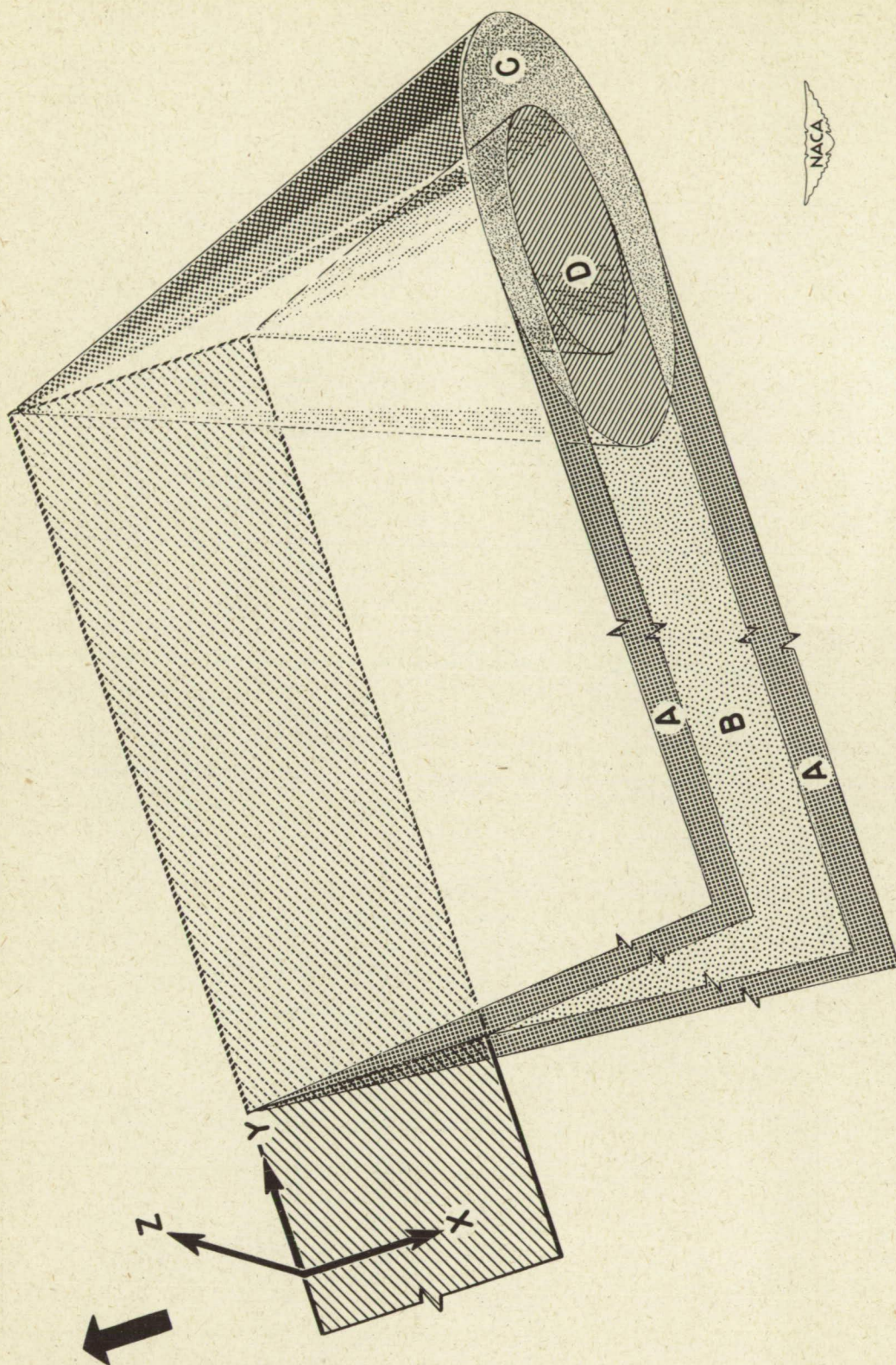


Figure 1. - Regions considered in examples for rectangular wing at supersonic speeds. Coordinate system shown is used in entire analysis; origin at leading edge of center section.